Proposed problem to Mathematical Reflections

Prove that

$$\frac{\sin x}{x} + \frac{(\sin x)^4}{x^4} \geq 2\cos x + \cos^4 x \sin^4 x, \quad 0 \leq x \leq \pi/2$$

for all $0 < x \leq \frac{\pi}{2}$